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ORIGINAL PAPER

# Taximeter verification with GPS and soft computing techniques

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**Abstract** Until recently, local governments in Spain were using machines with rolling cylinders for verifying taximeters. However, the condition of the tires can lead to errors in the process and the mechanical construction of the test equipment is not compatible with certain vehicles. Thus, a new measurement device needs to be designed. In our opinion, the verification of a taximeter will not be reliable unless measurements taken on an actual taxi run are used. GPS sensors are intuitively well suited for this process, because they provide the position and the speed with independence of those car devices that are under test. But there are legal problems that make difficult the use of GPS-based sensors: GPS coordinate measurements do not match exactly real coordinates and, generally speaking, we are not given absolute tolerances. We can not know whether the maximum error is always lower than, for example, 7 m. However, we might know that 50% of the measurements lie on a circle with a radius of 7 m, centered on the real position. In this paper we describe a practical application where these legal problems have been solved with soft computing based technologies. In particular, we propose to characterize the uncertainty in the GPS with fuzzy techniques, so that we can reuse certain recent algorithms, formerly intended for being used in genetic fuzzy systems, to this new context. Specifically, we propose a new method for computing an upper bound of the length of the trajectory, taking into account the vagueness of the GPS data. This bound will be computed using a modified multiobjective evolutionary algorithm, which can optimize a fuzzy valued function. The accuracy of the measurements will be improved further by combining it with restrictions based on the dynamic behavior of the vehicles.

**Keywords** Fuzzy systems · Genetic algorithms · Vague data · Fuzzy fitness function · GPS · Metrology

## 1 Introduction

From an engineering point of view, measuring the length of a path covered by a vehicle using the global positioning system (GPS) may seem an easy task using position data, speed data or both. Nevertheless, if these measurements are to be used for legal purposes, the situation is different. The measurements provided by a GPS receiver have a vague nature, and there might be large deviations between the actual position and the sensed coordinates of the vehicle. These deviations are infrequent, although they are possible, and this fact can invalidate the use of GPS in some applications.

This kind of behavior (i.e., an instrument having high accuracy most of the time, but also a chance of its accuracy being unacceptable) is not an exclusive property of GPS. Many other sensors used for legal purposes (for example, the radar) show this problem, albeit to a lesser degree. However, these sensors are routinely used because, in practical circumstances, if the probability of a measure being not allowable is very low, the evidence it provides is admissible. In other words, if we are able to provide an

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estimation of the length of a path which is conservative enough, so that the probability of the estimation being shorter than the actual length is negligible, we can use GPSs for legal purposes. Besides, the statistical model of a GPS is different from that of a radar in that it contains some different confidence intervals for the position of the receiver, given at different levels. If we honor the accuracy given by the lowest levels, then the measurements will be too coarse and, if we use the highest levels, the probability of the estimation being wrong will be too high. Therefore, we want to simultaneously process all these confidence intervals and find a balance between the accuracy of the measurement and its probability of being valid.

As we will detail later, such a family of confidence intervals matches some recent interpretations of a fuzzy set (Couso et al. 2001). These interpretations are being actively used in another Soft Computing field, that of learning of fuzzy rules from low quality data with genetic algorithms (Sánchez and Couso 2007) [in short, low quality data-based genetic fuzzy systems (GFS) or LQ-GFS] or in industrial applications (Otero et al. 2008). In this paper, we will model each measurement of the GPS, along with its family of confidence intervals, with a fuzzy set. Then, we will compute an upper bound of the length of the path. This will be derived from the membership function of the fuzzy estimation of the length. Previously, we will pre-process the data in order to remove outliers and redundant samples. This preprocessing will be based on certain heuristic rules which will be explained later, and involves the use of a genetic algorithm. Since the objective function is fuzzy-valued, we will use a specially crafted multicriteria genetic algorithm (GA), which is able to optimize fuzzy-valued fitness functions. Again, this genetic algorithm will be taken from the LQ-GFS field, where such kind of GAs are used to solve the aforementioned problem, finding fuzzy rules from imprecise data. In this respect, this paper follows the guidelines given in (Sánchez and Couso 2007), and applies the algorithms defined in (Sánchez et al. 2006; Casillas et al. 2001) and (Sánchez and Couso 2007) to a new research field. Plain fuzzy logic has been applied successfully to some problems that bear some relationship with this paper, in the area of multisensor fusion we refer the reader to Guixing et al. (2006). An example of improving the precision of GPS raw data with the aid of fuzzy logic can be found in Mosavi et al. (2002). Examples of the use of fuzzy logic in the context of navigation of different vehicles can be found in Zhao et al. (2007) and Naranjo et al. (2007).

The structure of this work is as follows. In the next section, we describe the problem to be solved. Then, in Sect. 3 how GPS measurements are obtained is detailed. We also explain the imprecise nature of GPS measurements, and how they can be interpreted as fuzzy data. The description of the proposal is set out in Sect. 4, where the filtering process and the issues regarding LUB computation are detailed. Deterministic (Sect. 4.2) and randomized (Sect. 4.3) algorithms for computing the LUB are given in the same section. In Sect. 5 details about the genetic algorithm used to filter the data are given. In Sect. 6 numerical results are shown. Finally, conclusions and future work are presented.

## 2 Problem statement

Taxi fares in Spain are regulated by local governments. Each time the fares are changed, the taximeters must be calibrated again and verified. One of the tasks to be performed in the Spanish VTSS is the testing and verifying of the taximeters in the taxicabs. The fares depend on two variables: the speed of the taxi during the service, and the length of trajectory covered during the service.

Since 1990, the Metrology and Models group at Oviedo University has been responsible for the design of the equipment and devices needed for the verification of taxis in Asturias, Spain. Currently, the verification of the taximeters is being carried out by means of a machine with rollers. The drive wheels are placed on the rollers, whose speed is regularly sampled. The test lasts a few minutes, while the driver must be assisted by a VTSS technician.

Once the test is over, the fare in the taximeter is compared to the fare in the machine with rollers. If the fare showed in the taximeter display is not 10% higher than the one calculated by the machine with rollers, then the taximeter may be used.

The total distance in the simulated run is computed by multiplying the circumference of the roller by the number of turns, and the linear speed is estimated from the angular speed of the rollers. However, the rollers have a relatively small radius, and the tire deforms differently over the rollers than over a flat surface. The difference between the actual and the theoretical radius means that tires appear to be smaller than they are for the system. Moreover, this error depends on the tire condition and the weight of the vehicle, making the whole test unreliable.

We intend to introduce a new portable system, that uses a GPS sensor to sample the position and the speed of the taxi at regular intervals. The new test would imply a significant saving for the VTSS station, because the technician is no longer needed and the cabin where the rollers are installed would be freed for other uses. Because of cost reasons, we also want to use cheap, consumer grade GPS. Price matters, because each station must acquire between 10 and 20 devices; otherwise the queueing time needed would not be acceptable.

Unfortunately, there are legal problems that complicate the use of GPS measurements. It is well known that data collected by a consumer grade, nondifferential GPS, is imprecise at submeter level. GPS manufacturers usually report the accuracy of a receiver in terms of a CEP value (see Sect. 3 for details), but with no reference to the number of satellites used, their relative positions, ionospheric influence or multipath; these sources of error must be taken into account to calculate the tolerance of the measurements in a given fix.

Unless we are able to bound the tolerance in our estimation of the length of the trajectory and the speed of the vehicle, we will not be able to legally reject a taximeter. This is the same problem that happens, for instance, when speed penalties are applied by a highway radar: we can not penalize a vehicle whose speed is higher than the limit unless we also know (a) the tolerance of the radar, and (b) that the measured speed surpasses the limit by more than that tolerance. In any other case, we must assume that the driver (and, conversely, the taxi owner) has not committed an offence.

Therefore, it is difficult to homologate a GPS-based device, because we can not assess its absolute accuracy, e.g., the tolerance may be 5% for a certain route and 7% for a different route. If we knew that the tolerance is always under 10% (which is the legal limit) we could homologate the device, but we can not assert that. Thus, in this paper, we propose a device that not only produces an estimation of the length of the trajectory, but it also computes an upper bound of this length. In this way, once the taxi has finished the run, we can know whether the tolerance of the measurement has been under the legal limit or not, and repeat the test if needed.

### 2.1 Legal constraints and statistical decisions

Let us suppose we have a measurement device which, given a taximeter with an unknown error $e$, produces an estimation $\widehat{e}$ of its error. We will assume that the device is unbiased, i. e. $E(\widehat{e}-e)=0$. Therefore, we could define the trivial decision rule that follows:

$$D_0(\widehat{e}) = \begin{cases} \text{Accept} & \text{If } \widehat{e}\leq 0 \\ \text{Reject} & \text{otherwise.} \end{cases} \quad (1)$$

However, any measurement device will have a tolerance $\epsilon$: this means that $\widehat{e}-\epsilon\leq e\leq\widehat{e}+\epsilon$ with a very high probability and, conversely, that the probability $p(|\widehat{e}-e|>\epsilon)$ is near zero. This tolerance has legal implications. Suppose, for instance, that we reject a taximeter because we estimate that its error is $\widehat{e}=5\%$, and the tolerance of our device is $\epsilon=7\%$, which is higher than this error. The taxi owner could argue that there is a chance that the true error of the taximeter is less than or equal to 0, and have our rejection revoked. In short, we can not reject a taxi unless the estimation of the error is higher than the tolerance, thus we are sure that the taximeter is incorrect with a high probability.

In Spain, to prevent taxi users from fraud, the charged fare must be less than 10% higher than the true fare, and therefore we can not homologate a device with a tolerance higher than this value. It is remarked that the tolerance of a GPS device depends on many factors (geometry or constellation of the satellites, shape of the trajectory, speed, etc.). As we have mentioned in the preceding section, we can not certify that all measurements taken with a certain GPS device will be more accurate than 10%. However, we will show in this paper that we can determine whether the tolerance of a particular measurement has been within the legal margins. This is the main objective of this paper.

We have also mentioned that the legal problem is similar to that of using a radar for measuring the speed of a vehicle. But, the legal assumption of innocence, that most drivers would be glad to accept if accused of surpassing the speed limit, benefits the taxi owner.

The legal decision rule is

$$D(\widehat{e}) = \begin{cases} \text{Accept} & \text{If } \widehat{e}\leq 10 \\ \text{Reject} & \text{otherwise} \end{cases} \quad (2)$$

which, from a statistical point of view, is not fair. Let $p(x)$ be the probability of the error of a taximeter being $x$.

On the one hand, we can reject a correct taximeter with a probability

$$P(\text{Reject} \mid e\leq 0) = \frac{p(\widehat{e}>10\cap e\leq 0)}{p(e\leq 0)}. \quad (3)$$

On the other, we will pass an incorrect taximeter with probability

$$P(\text{Accept} \mid e>0) = \frac{p(\widehat{e}\leq 10\cap e>0)}{p(e>0)}. \quad (4)$$

For $D$ to be fair, we need both errors to be the same. However, they are not. If the tolerance of the device is lower than 10%, then $p(\widehat{e}>10\cap e\leq 0)$ is near zero, thus $P(\text{Reject} \mid e\leq 0)$ is negligible, but $P(\text{Accept} \mid e>0)\geq p(0<e<10-\epsilon)/p(e>0)$, which is rather high. According to our own experience, Spanish taxi drivers calibrate their taximeters to obtain their maximum legal advantage, i.e., it is by far more frequent that a taxi has an error near 10 than an error lower than 0.

### 2.2 Lower upper bound of a trajectory

In order to obtain the highest number of valid measurements (i.e., those for which the tolerance is lower than 10%), we are interested in knowing the shortest trajectory whose length is known to be longer than the actual path, given a set of imprecise coordinates of the vehicle.

Therefore, in this work, it is proposed to calculate the lower upper bound (from now on called LUB) of all possible trajectories that are compatible with the measurements given by the GPS.

Due to the imprecise nature of input data, a new method for establishing the LUB is presented. In doing so, input data is represented as fuzzy data. This data is filtered in order to produce the smallest subset of coordinates that induces a multipolygonal that covers the input data as much as possible. Finally, that filtered data is fed to a deterministic algorithm for computing the upper bound of the length of the trajectory.

The filtering process is the most complex part of the procedure. It involves solving a multicriteria optimization problem, for which we will use the genetic algorithm NSGA-II (Deb et al. 2000; Deb and Goel 1993).

## 3 The vague nature of the GPS measurements

The term global positioning system (GPS) (Hofmann-Wellenhof and Collins 2004) refers to a set of devices (satellites and receiver) working together to get a fix (the position) of the receiver. The receiver can receive some signals from the satellites and compute a set of measurements: longitude, latitude, altitude, number of satellites in use, time, etc. Each signal received from a satellite contains information about the time that the signal takes to travel from the satellite to the receiver.

Using signals from four satellites, a GPS receiver can compute the three 3D coordinates and data for time correction (Mohinder et al. 2007). If more satellites are in view, they can be used to improve accuracy. For example the four best positioned can be selected to compute position and time. Another way to improve accuracy is overdetermining the equation system (Lachapelle and Ryan 2000), trying to minimize the errors due to perturbations of the satellites signals when crossing the atmosphere, satellite ephemeris deviation, satellite clock errors, receiver errors and multipath (Hofmann-Wellenhof and Collins 2004). As a rule of thumb, the higher the number of satellites the better the accuracy. But even with a high number of satellites in use (12–16) the geometry or constellation of the satellites must be taken into account to estimate the fix accuracy. This is done using DOP (dilution of precision), a measurement of the probability of the effects of the constellation on the fix accuracy (Langley 1999), a higher value of DOP indicates a weaker geometry of satellites. DOP has four components: PDOP (3D or spherical DOP), HDOP (latitude and longitude DOP), VDOP (vertical DOP) and TDOP (time DOP).

Under certain conditions, GPS measurement errors follow a bidimensional Gaussian distribution. When many satellites are available, that distribution can be regarded as circular (van Diggelen 2007). Because of this, consumer grade GPS receivers give an indication of their precision through a magnitude called Circular Error Probable (in following CEP): the radius within which 50% of the horizontal position solution will fall and it is centered at the true position.

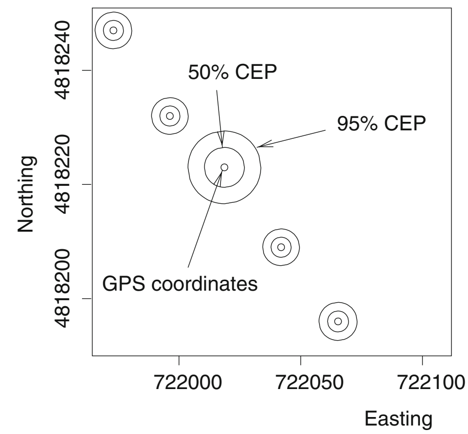


**Fig. 1** Real GPS measurements, the CEP value changes from one measurement to another

CEP can be computed from the standard errors of the estimated coordinates with Eq. 5 (Langley 1991; Strang and Borre 1997).

$$CEP = 0.56\sigma_x + 0.62\sigma_y \qquad (5)$$

The CEP at 95% probability is also known as R95 and can be obtained multiplying by 2.08 the 50% probability CEP.

In Fig. 1 a real example showing how the CEP could vary between consecutive measurements can be seen.

Consumer grade GPS do not send information related to the standard errors. HDOP values are available from the standard National Marine Electronics Association (NMEA) protocol used in most of the GPS receivers and this magnitude accounts for the impact of constellation geometry in horizontal accuracy. Thus, an empirical estimation of CEP/HDOP relationship must be carried out (Dussault et al. 2001; Cressie 1991). From the definition of CEP, this is easily done from a sample of GPS coordinates taken in a given location. For each HDOP value, the subset of GPS obtained under that value is extracted from the whole data. Then the smallest circle that covers 50% of the points is the CEP at that probability. The procedure for the 95% CEP is analogous.

### 3.1 A fuzzy representation of GPS data

In the context of imprecise probabilities, a fuzzy set could be seen as a set of tolerances. Each tolerance is assigned a confidence rate, and the lower the tolerance the lower the confidence rate (Goodman and Nguyen 1985). In particular, given an incomplete set of confidence intervals of a random variable, it is possible to generate a random fuzzy variable for which α-cuts are confidence intervals of rate $(1 - \alpha)$ (Couso et al. 2001). We will use this representation and perform a multi-level calculation of the LUB.

In the case of the measurements obtained from a GPS unit, two confidence intervals are given at 50% and at 95%. Using the procedure explained in this section, a value of CEP could be calculated for each probability value. It can easily be seen that the higher the probability value the higher the CEP value. The physical meaning of this fact is simple for the GPS measurements: the higher the confidence rate needed for determining the real position from which the GPS measurement was taken, the higher the CEP value.

## 4 Calculation of a LUB using fuzzy data

The GPS measurements are sampled at equally spaced time intervals. Each measurement is a fuzzy set, as stated before, whose α-cuts are circles centered on the GPS coordinates. Therefore, each circle is a confidence interval for the true coordinates of the taxi when the measurement was taken. In Fig. 2 some simulated GPS measurements and trajectories are shown. The position where the measurement was taken is in the real trajectory—continuous line—and it can be inside or outside the respective circle of radius CEP. The trajectory using the GPS coordinates is drawn using a dashed line. A trajectory totally compatible with the GPS measurements is also drawn as a dotted line. Notice that the lengths of these trajectories are different, but all of them are compatible with the measurements of the GPS. Thus, to know the accuracy of the measurement, we want to compute the lower upper bound of the lengths of all the paths which have all of their vertexes inside the circles (for each confidence level). This bound is the largest trajectory compatible with the GPS measurements.

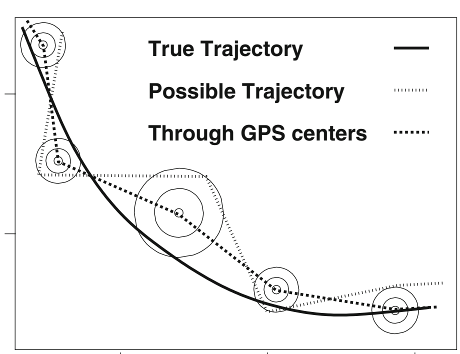


**Fig. 2** Simulated example where the differences between the true trajectory and the trajectory through the GPS coordinates are shown

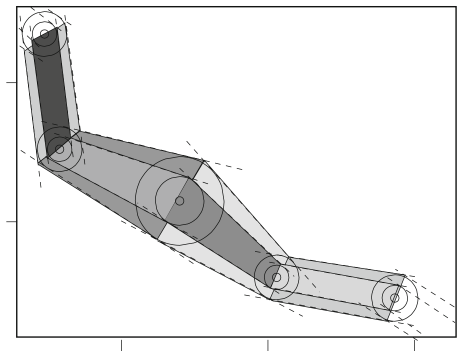

**Fig. 3** Simulated example showing how a polygonal that covers the fuzzy input data from GPS could be built

Observe that the LUB is infinite unless we introduce some constraints. The main assumption in this paper is that *changes in the direction of the vehicle between two samples are small*, thus we can approximate the trajectory between two consecutive samples by a straight line.

We will define a polygonal chain that covers the fuzzy data, by finding the outside tangents to the CEP for each α-cut, and then computing the cross points of the corresponding tangents between two consecutive fuzzy points (see Fig. 3). Observe that

(1) The longest segment contained in each polygon of four sides, a quadrilateral, is one of its diagonals.
(2) The longest path contained in two adjacent quadrilaterals always comprises two of these diagonals.
(3) The longest path contained in three adjacent quadrilaterals is also composed of diagonals, but they might not be the longest diagonals of each quadrilateral.

This means that the LUB is defined by a list of vertexes, but it is not immediate to find out which vertexes from the polygonal chain should be chosen to define the path. Later in this section it is explained how we solve this problem, however let us first introduce first some preprocessing algorithms that will improve the accuracy of the final measurement.

### 4.1 Preprocessing the data

The accuracy of the measurements can be improved if some collinear points are joined, and some of the worst GPS fixes are discarded. If the sampling rate is high enough, our hypothesis (straight trajectory between samples) still holds after performing these two changes, and we can safely preprocess the data before computing the estimation of the length and the LUB.

Given a level value α, the fuzzy input data is represented as a circle centered on the coordinates, and whose radius is the CEP at probability $(1 - \alpha)$. We have mentioned that, for each α-cut value, the data comprises a set of circles, and also that the two outside tangents of two consecutive circles, and the cross points of the tangents to every three consecutive circles define a polygonal chain. The objective of the preprocessing is to obtain a reduced polygonal chain of fuzzy sets which still contains the LUB for each level α. If crisp data were used, polygonal chain simplification has been studied in (Estkowski and Mitchell 2002; Hershberger and Snoeyink 1992; Buzer 2009; Drysdale et al. 2008; Chen et al. 2005; Gudmundsson et al. 2007). For fuzzy data, to our knowledge, the most similar work in the available literature is that presented in (Anile et al. 2000), where fuzzy data from a geographical data base are used to reconstruct 3D images by means of fuzzy B-splines (de Boor 1972). Further to the approach in this paper are (Li and Chen 1999; Quddus et al. 2006).

Our process of computing the LUB is a three step procedure: the first step is filtering by collinearity, then filtering by a multiobjective algorithm, and finally using a deterministic algorithm for determining the largest polyline in a polygonal chain; this is done for each α-cut. These filtering stages are described in the subsections that follow, and the deterministic algorithm is explained in Sect. 4.2.

#### *4.1.1 Filtering by collinearity*

For each α-cut a polygonal chain is obtained, as explained before. If for three consecutive circles, both pairs of outside tangents are parallel, then the quadrilateral defined by the first two circles is contained in the quadrilateral defined by the first and third circles, and the intermediate circle could be filtered. Further removals of points are limited by the sampling rate, i.e., if too many points are filtered out, we can not assume the trajectory is straight between the first and the last one.

#### *4.1.2 Filtering spurious data*

Spurious data are input points where the error is abnormally high. We can remove those points where the fix was not accurate enough, provided that we do not discard a significant number of points, i.e., a fraction $1 - \alpha$ of the vertexes of the reduced polygonal must be in the unfiltered path, for each level α. In Fig. 4 the process is illustrated for a given α-value.

Filtering points reduces the area of the polygonal chain, so the LUB will be smaller too, and this is not a desired effect. We do not want to filter representative points. Maximizing the percentage of covered data, while filtering the outliers, are objectives that counteract one another. We will use a multicriteria genetic algorithm to optimize the filtering, as we will explain in Sect. 5.

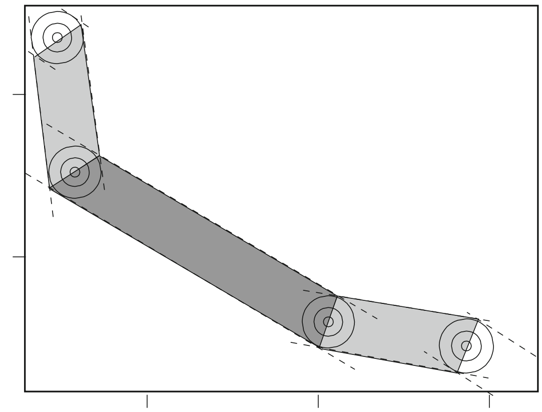

**Fig. 4** The simplifying process over a synthetic example: filtering a point reduces the area of the polygonal chain, but the remaining points must resemble the trajectory

#### *4.1.3 Dynamic behavior of the vehicle*

The filtering process can be further improved if we assume that the vehicle has inertia and some trajectories contained in the polygonal chain are not feasible (Chen et al. 2005). For example, think of a vehicle moving at a certain speed. There exists a maximum angle the vehicle could turn without risking its security. Moreover, for each speed value there exists a maximum angle, lower than that given due to security reasons, which is comfortable for the vehicle passengers.

Therefore, we can introduce a second hypothesis in our analysis: *driving a taxi must be comfortable*. The maximum angle of turn is a function of the speed of the vehicle: the greater the speed, the shorter the maximum angle of turn. In determining the angle of turn at each point, the one before and the one after fuzzy points are used. For an α-cut value the fuzzy points are circles. By means of the tangents the polygonal chain for the three fuzzy points could be defined. The largest trajectory included in such polygonal chain is one of the four possible polylines that goes through the vertexes of the polygonal chain. Once the largest trajectory for this three points polygonal chain has been found, if the angle that the segments of the largest trajectory define is larger than the maximum angle of turn at current speed, the trajectory is not considered as a candidate for its length being the LUB.

### 4.2 Deterministic longest path estimation

Once the data are preprocessed, we evaluate its LUB with a deterministic algorithm, that we explain in this section.

For each α-cut of the chain, we get a polygonal set constructed with trapezoids, as can be seen in Fig. 5. The motion direction is indicated by a thin dashed arrow. Each trapezoid vertex is denoted with a pair of integers, those at

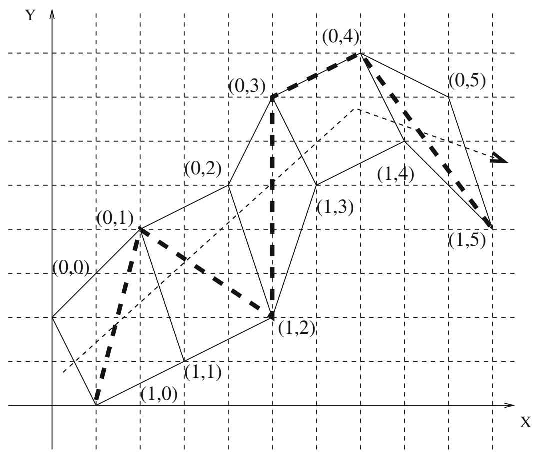


**Fig. 5** Example of longest path estimation following the algorithm explained in Sect. 4.2

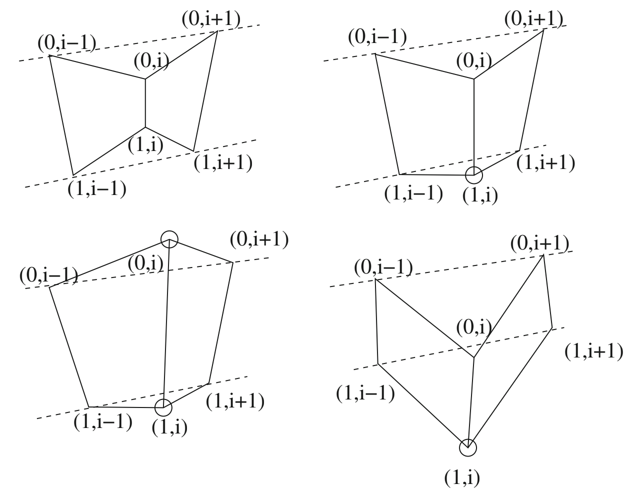


**Fig. 6** Possible relative positions of vertexes and lines between prior and next vertexes. This is useful in the determination of convex and nonconvex vertexes

the left of the arrow have zero at first, those at the right have one at first. The other number is the step in the motion sequence. The longest path at each step $i$ goes through $(0, i)$ vertex or $(1, i)$ vertex. The set of vertexes that defines the longest path, can be computed by exhaustive exploration of all possible combinations, but this is very expensive in terms of computational cost and proved impracticable in a realistic trajectory with 100 points, for instance. This problem has been studied in the area of Computational Geometry and is related with Longest Path with Forbidden Pairs (Berman and Schnitger 1992), that is NPO PB-complete.

Due to this fact and given that in a realistic trajectory the changes of direction and the changes in distance between left and right vertex are limited due to the dynamics of the taxi, the geometry of the road and GPS behavior, we use a heuristic that is linear in time with the number of vertexes. The heuristic is based on the selection of convex vertexes: when a vehicle turns, the longest path goes through the exterior of the trajectory curvature. The convexity of a vertex is analyzed using the straight lines that rely on previous and following vertexes. The possible relative positions of the central vertex can be seen in Fig. 6, where convex vertexes are marked with a small circle and the lines that pass through vertexes $(0, i-1), (0, i+1)$ and $(1, i-1), (1, i+1)$ are drawn. From left to right and top to bottom, if both vertexes are between the lines, both are concave. If only one is outside the lines, it must be convex. If both are outside the lines, both may be convex (left) or one may be concave and the other is convex. In both cases, if the farthest one from the nearest line is chosen, then it is convex. The heuristic is as follows: the first segment of the longest path goes from a convex vertex in step 1 to the vertex at step 0 that gives the maximum segment length. From step 1 to the one before the last, the path goes through:

- If there is only a convex vertex, through this vertex.
- If there are two convex vertexes, through the farthest one.
- If there are no convex vertexes, through the farthest one.

The last segment ends in the farthest vertex from the previous one.

In Fig. 5 the path computed with this heuristic is marked with a thick dashed line. The first segment goes from $(1, 0)$ to $(0, 1)$ because $(0, 1)$ it is convex and the distance to $(0, 0)$ is shorter. Then the longest path continues to $(1, 2)$ because it is the only convex. The same happens with $(0, 3)$ and $(0, 4)$.

Finally, the path ends in $(1, 5)$ because it is farther from $(0, 4)$ than $(0, 5)$).

## 4.3 Randomized longest path estimation

There is an alternate implementation for the method proposed in the preceding section. Let us suppose that we superimpose a grid on the chain of circles (see Fig. 7) and compute the whole set of lengths obtained by the selection of one point of the grid from each fix. In order to compute the maximum length from the obtained trajectories, a backtracking algorithm is impracticable due to the high number of points present in real trajectories. However, we can uniformly sample a number of trajectories and compute a Montecarlo estimate of the probability distribution of the lengths that arise. The mean value of this distribution would be our estimation of the length, and we can also produce a confidence interval for the mean, from which to calculate our tolerance.

To build a sample trajectory, we have implemented the procedure that follows: for each fix, with a probability 0.95,

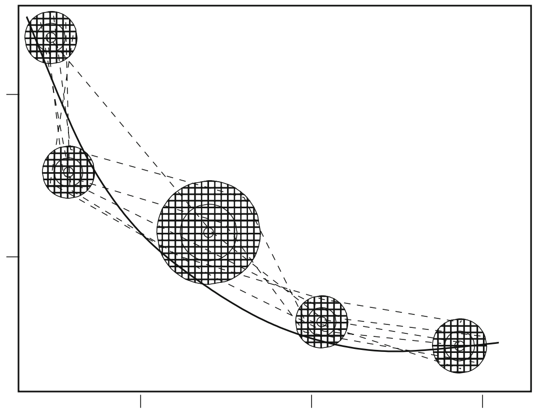

**Fig. 7** Discretization of GPS measurements and exhaustive exploration of the lengths of all the trajectories obtained from all the possible combinations of discretized points. Only a fraction of the possible trajectories is shown

a point of the grid which is inside the CEP at that level is randomly selected (thus with a probability 0.05 the point is selected from outside the corresponding CEP). The length of the polygonal that joins all these points is stored and the whole procedure repeated a high number of times. The sample distribution of these lengths is finally used to compute the sample mean (which is an estimator of the length that the taximeter should have produced) and a confidence interval for it.

Unfortunately, there are some issues with this method. In particular, the range of the sample distribution of the lengths is bounded by the maximum sample length, which in general is not the maximum length of a compatible path. As a matter of fact, the maximum of the sample would be a more reliable estimator of the LUB for our purposes. Since there are not clear improvements neither in speed nor in accuracy with respect to the method in the preceding section, Montecarlo analysis is not considered in this work.

### 4.4 Predetermined trajectories and maps

A trivial improvement of the accuracy could be obtained if we restricted ourselves to a known route, and adjust the GPS coordinates so that each position is replaced by its nearest point in the center of the road. However, this method has been rejected by the experts in the certification agency that will homologate the device. The reason given was that all the measurements have to be reproducible. There will be points with known latitude and longitude where the certification agency will position the devices and check that the tolerance of the measurements is within the range. This kind of certification is not compatible with dynamic changes in the coordinates, because the nearest point of the road may well be out of the circle defined by the CEP, making the determination of the LUB useless.

However, nothing prevents using the map as a constraint, i.e., we know that the car is in the intersection between the road and the confidence interval produced by the GPS. The coordinates of the whole road and not only its center are needed, though. This method has not been implemented either as we have not measured the width of the road, nor do we have access to a cartography of the route with legal validity. This last point is important, because any error in the maps would void all the measurements taken with the devices. Nevertheless, observe that, even if we decided to include this information in the future, it would not alter the computations we present in this paper.

## 5 Genetic filtering of the fuzzy data

In this section the details of the codification and operators used in the genetic algorithm which has been used to filter the data are given. A multiobjective genetic algorithm has been used in the filtering process. Specifically, the multiobjective genetic algorithm used in this work is the well known NSGA-II (Deb et al. 2000; Deb and Goel 1993). This algorithm is outlined in Fig. 8.

### 5.1 Codification of an individual

Each individual is a subset of the chain of fuzzy points, codified as an array of booleans. That is to say: input data is a series of timely ordered fuzzy points. Each one of the fuzzy input points has a boolean value associated for each individual. When the boolean value for a fuzzy point is set to *true* then that fuzzy point is included by the individual. When the boolean value for a fuzzy point is set to *false* that fuzzy point is filtered out.

To generate an individual, a probability threshold $p$ is given, and each fuzzy point in the vector of input fuzzy data is included with probability $p$. The origin and the end of the taxicab run must always be included.

```
R_t = P_t ∪ Q_t
F = fast-non-dominated-sort(R_t)
P_{t+1} = φ,  i = 1;
while |P_{t+1}| + |F_i| > N
    crowding-distance-assignment(F_i)
    P_{t+1} = P_{t+1} ∪ F_i
    i = i + 1
end while
Sort(F_i, ≺_n)
P_{t+1} = P_{t+1} ∪ F_i[1 : (N − |P_{t+1}|)]
Q_{t+1} = make-new-population(P_{t+1})
t = t + 1
```

**Fig. 8** Pseudocode of the NSGA-II algorithm

### 5.2 Multiobjective fuzzy fitness function

Observe that both the area of the polygonal chain and the percentage of covered fuzzy data by the polygonal chain are fuzzy functions. We want to mimimize the area of the defined polygonal chain and to maximize the percentage of covered fuzzy data. Following (Sánchez and Couso 2007), we will not defuzzify the objectives, but use a fuzzy valued fitness function to assess the quality of the filtering.

For a genetic algorithm being able to solve a fuzzy valued optimization, we can define a total order between the fuzzy values of the fitness function (Abbasbandy and Asady 2006; Mitchell 2006; Tran and Duckstein 2002; Sheen 2006; Sun and Wu 2006). In particular, we need to sort fuzzy numbers, which some authors think is inconsistent with most definitions of total order between fuzzy sets (Yeh and Deng 2004; Wang et al. 2005). The use of a weaker (partial) order can also be made compatible with a tournament-based selection in conventional genetic algorithms. In this context, in (Jahanshahloo et al. 2004; Wang et al. 2005) an interval representation is used, and in (Ganesan and Veeramani 2006) another partial order relation is proposed, restricted to trapezoid membership functions. This last solution can not be applied to our problem, either, as there is neither knowledge nor restrictions about the membership functions type and certainty distributions. Also, different approaches for evaluating the Pareto dominance using fuzzy fitness functions have been proposed. In the use of fuzzy rules (Youssef et al. 2000) for determining the dominance of one individual with regard to another is proposed. Similar works are documented in (Trebi-Ollennu and White 1997; Kiyota et al. 2000). In this work we have decided to use our own implementation of the NSGA-2 algorithm for fuzzy data, which is described in Sánchez and Couso (2007), Sánchez and Couso et al. (2009). We have used an imprecise probabilities based ranking, in combination with the definitions of nondominated sorting and crowding distance explained therein.

### 5.3 Genetic operators

The definitions of the crossover and mutation must reduce the number of vertexes in the population.

- *Crossover*. Given two parents $A$ and $B$, the offspring are two new chains $C$ and $D$ such that $A \cap B \subseteq C$ and $A \cap B \subset D$; a vertex $v \in A - B$ has a probability $p^+$ of being in $C$, and a vertex in $B - A$ has a probability $p^-$ of being in $C$, where $p^-$ is much lower than $p^+$. $D$ set is constructed in the same way.
- *Mutation*. This operator is defined as the random removing of a point of the chain, different from the first or last one. It is important to notice that neither the first nor the last fuzzy points will be included in the genetic operations because both trajectory ends must be included for all individuals.

## 6 Experiments and results

In this section the results of the experiments that support the claims in Sect. 2 are shown. Therefore, the experiments are designed to:

(1) Evaluate the accuracy of the proposed method in a realistic scenario using synthetic data.
(2) Compare both methods in real situations.

Because of this, two batches of experiments were performed:

(1) Simulated GPS data acquisition from synthetic data: the run of the vehicle is simulated and the GPS measuring process is also simulated adding random noise to mimick the properties of a GPS sensor. Under these controlled conditions the percentage of error between the LUB and the known length of the path is estimated.
(2) Compared performace in real world situations: a real circuit is measured with a certified ISO-9002 odometer and the reported length is compared with that obtained with the proposed method. The run with the same length is measured using the same vehicle and the roller machine, in order to compare the obtained results.

In the following sections these experiments are detailed.

### 6.1 Synthetic data

It was decided to evaluate our algorithm in a realistic path that covers the situations usually found when the VTSS test of a taxi is carried out. Since we need to know the true length of the path, these datasets must be synthetic. The data includes several turns, accelerations and decelerations and changes in CEP. GPS longitude/latitude coordinates were translated to Universal Transverse Mercator northing/easting coordinates in order to make distance calculations between GPS fixes easier (Snyder 1982). With this system, points on the Earth's surface are projected onto an equally spaced planar metric grid, therefore the distance between fixes is the usual Euclidean one.

The trajectory is sampled once each second. At each location, a random value ranging from 4 to 8 m is taken as the CEP at 95% probability, being suitable values for a consumer grade GPS receiver. The uncertainty in the GPS measurements is simulated using the following procedure: with a probability of 0.95, a point is selected whose

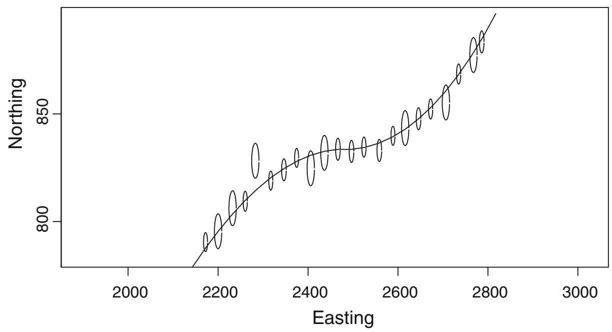


**Fig. 9** Example of GPS generated data along with the true trajectory. As can be seen, some of the fixes CEP do not intersect the true trajectory. The majority of the points of the trajectory are inside the CEP at 95%

distance to the real one is shorter than the CEP, and with probability of 0.05 an outlier is introduced. Part of the generated data is shown in Fig. 9. GPS measurements are represented with circles (actually ellipsoids in Fig. 9, due to scalling issues) with a radius equal to 95% CEP and the original trajectory with a continuous line. As can be seen, 95% of the circles intersect the trajectory, but none of their centers are in the actual trajectory.

We perform three experiments in order to test if the tolerance computed from the LUB is lower than the legal margin or not.

If the true length of the trajectory is known, the tolerance is

$$\epsilon = (\mathrm{LUB} - \mathrm{length})/\mathrm{length}. \quad (6)$$

The synthetic trajectories generated for this batch of experiments were:

- A first 120 points dataset, with smooth turns, similar to a conventional road.
- A second 120 points dataset, with stronger turns, also similar to a conventional road.
- A third 200 points dataset, similar to test circuit, with a mixture of strong and smooth turns in all possible directions.

Several tests were also carried out to assess the genetic filtering. This was launched with the parameters shown in Table 1. Firstly, and for both trajectories, ten runs of each of the multiobjective algorithm were done, without restrictions based on the dynamic behavior of the vehicle. Finally, a second set of ten runs were done with the multiobjective algorithm using the same parameters but assuming that the angular velocity of the vehicle had a speed dependent bound.

### *6.1.1 Case study I*

The true length of the first trajectory is 3,228.6 m. The distance through the GPS fixes is 3,238.5 m. Taximeters

**Table 1** Parameters used for NSGA-II algorithm

| Parameter | Value |
|---|---|
| Number of generations | 1,000 |
| Number of individuals | 100 |
| Number of iopulations | 1 |
| Minimum percentage of fuzzy points covered by each individual | 0.85 |
| Probability $p^+$ for genetic crossover | 0.5 |
| Probability $p^-$ for genetic crossover | 0.1 |
| Crossover probability | 0.7 |
| Mutation probability | 0.1 |

**Table 2** Results from 10 runs of NSGA-II without dynamic analysis

| Dataset | True length | Measured length | LUB estimation standard deviation | LUB estimation mean |
|---|---|---|---|---|
| 1 | 3,228.57 | 3,238.521 | 16.95 | 3,499.48 |
| 2 | 2,741.30 | 2,696.487 | 43.55 | 3,192.51 |
| 3 | 9,337.78 | 9,364.49 | 43.55 | 9,404.68 |

**Table 3** Results from 10 runs of NSGA-II with dynamic analysis

| Dataset | True length | Measured length | LUB estimation standard deviation | LUB estimation mean |
|---|---|---|---|---|
| 1 | 3,228.57 | 3,238.521 | 0.52 | 3,243.45 |
| 2 | 2,741.30 | 2,696.487 | 10.88 | 2,798.01 |
| 3 | 9,337.78 | 9,364.49 | 10.88 | 9,353.85 |

which mark more than 3,228.6 + 10% = 3,551.4 will be rejected. Observe that, in practical circumstances we do not know the actual length of the path and we will reject those taxis that charge more than 3,238.5 + 10% = 3,562.4.

The mean LUB for this trajectory (first row in Table 2) is 3,499.5. Then, the expected tolerance is 0.084 (0.081, if we used the measured values to compute the tolerance). This result means that taximeters charging more than 3,551.4 can be safely rejected.

If we filter the data taking into account the dynamics of the vehicle, (see Table 3) the expected tolerance is 0.005 (0.002 if we used the measured length) which is much lower than the legal margin.

### *6.1.2 Case study II*

The length of the second trajectory is 2,741.3 m. This trajectory has stronger turns than the first, and we will see that the LUB will be less tight, given the deterministic procedure explained in Sect. 4.2. The distance through the


**Fig. 10** First version of the prototype of the data logger that is being developed for the verification of taximeters. An evolution of this device is shown in Fig. 11

GPS fixes is 2,696.5 m. If we repeat the analysis done in the preceding case (see second row of Table 2) we obtain that the mean LUB is 3,127.0 m. The expected tolerance is 0.141 (0.160), thus we can not reject a taximeter on the basis of a test carried out on this route.

The use of dynamic restrictions in the genetic filtering causes a significant improvement of the tolerance, which is now 0.021 (0.038,) thus we could legally reject taxis that charge more than 2,741.3 + 10% = 3,015.4 (2,966.2) using the same input data.

Observe that, given these results, we recommend that the VTSS station choose circuits with smooth turns, since they will surely produce a low rate of null verifications. In general terms, the use of convoluted paths is not advised with the system we propose here.

### 6.1.3 *Case study III*

The length of the third trajectory is 9,337.78 m. As stated before, this trajectory resembles a test circuit and has a mixture of stronger and smooth turns.

The distance through the GPS fixes is 9,364.49 m. If we repeat the analysis done in the preceding case (see third row of Table 2) we obtain that the mean LUB is 9,404.68 m. The expected tolerance is 0.007 (0.004).

Again, the use of dynamic restrictions in the genetic filtering causes a significant improvement of the tolerance, which is now 0.0011 (0.0017).

In this case, any taximeter that charges more than 9,337.78 + 10% = 10,271.56 can be legally rejected.

Note that the computed LUB is *under* the measured length but *over* the real length: when GPS measurements are accurate, the LUB is tighter.

## 6.2 Real-world measurements

For taximeter verification, the taxi owner is sent on a run. He/she must carry the target taximeter, the GPS and the datalogger device. Once the taxi comes back to the VTSS, the GPS measurements are loaded into the computer where LUB computing will be carried out. Only bi-dimensional measurements are to be taken into account as the losses of information using this representation are not significant: the trajectories range between 3 and 5 km, and the differences in altitude are a few meters. Each measurement includes the coordinates, the calculated CEP value for the desired $\alpha$-cuts and the HDOP. The prototype which is currently under development is shown in Figs. 10 and 11.

**Fig. 11** *Left* evolution of the prototype shown in Fig. 10 inside a commercial enclosure.The design is based on low cost automotive microcontrollers and a consumer-grade GPS receiver. *Right* the same prototype, with the enclosure opened showing the pcbs of the device

**Fig. 12** Aerial picture of the circuit used in the tests. In the left of the image, a portion of the road is shown between the building and an area covered with trees, where GPS measurements are difficult

The device was entirely designed at the Metrology and Models group at Oviedo University. It uses three Atmel AVR microcontrollers, and collects NMEA data from a San Jose Navigation FV-M5 GPS module. It incorporates a flash memory chip (Atmel dataflash) to store data, as well as a USB connection to program the controllers, send or receive data from a PC. Moreover, the device's got an LCD display and a bubble keyboard to control its operation.

In this section we have used a circuit surrounding the Campus de Viesques' buildings (Gijón, Asturias, Spain). The length of the circuit was measured using an ISO-9002 certified odometer. In Fig. 12, a picture of the circuit can be seen. The circuit comprises two long and two short straight paths, and four sections with sharp turns. The measured length was 1,093 m.

A direct comparison of the rolling machine against the other methods is not possible, so an indirect procedure was employed. We equipped a car with an odometer, that reported the distance travelled. Then we placed the same car in the rolling machine, and travelled 1,093 m, according to the same instrument. Lastly, we read the measurement produced by the rolling machine. This procedure was repeated 10 times.

In Fig. 13, the route recorded by the GPS in the analysed laps can be seen. There are large differences between the centers of the measurements between successive laps, especially in the left part of the circuit. This uncontrollable behavior results in a variation of the measured length of the same path, as shown in Table 4. This table collects the mean and standard deviations of the obtained measurements, using each method discussed in the paper: NSGA-II dynamical filtering, raw GPS coordinates, and cylinders. It is remarked that the variability of the results in the cylinders does not take into account the variability induced by changes in pressure of the tires or their wear. As can be seen, the variability of the proposed method and the roller machine are comparable and much lower than that obtained using unfiltered GPS data. Moreover, the obtained LUB is effectively an upper bound of the actual length and can be used to take the decision of when the test must be repeated or not. The $p$-values obtained from nonparametric Wilcoxon test are shown in Table 5. As can be seen, the differences between NSGA-II and the other methods are statistically significant at 0.05 level, even after Bonferroni adjustment (Hsu 1996) for a number of comparisons $n = 3$. In this case, the differences are significant if the obtained $p$-value is less than $\alpha/n = 0.0167$.

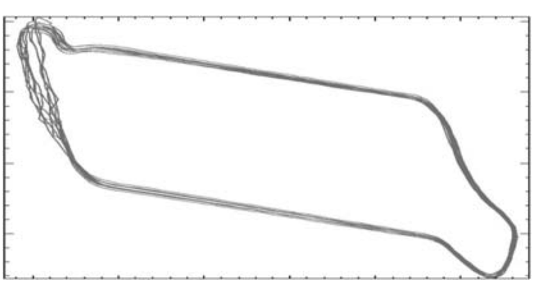

**Fig. 13** Path registered by the GPS, ten laps of the circuit depicted in the preceding figure. There are large differences between the laps, mostly in the left part of the circuit

**Table 4** Mean and standard deviation of the 10 measurements of the real path using NSGA-II filtering, unfiltered GPS coordinates and equivalent rolling cylinder machine run

| | NSGA-II | GPS | Rolling Mach. |
|---|---|---|---|
| Mean | 1,121.494 | 1,103.635 | 1,106.667 |
| Standard deviation | 7.665415 | 16.49395 | 5.532274 |

The measured length using a certified odometer was 1,093 m

**Table 5** $p$-Values of Wilcoxon test between each of the tested methods

| | NSGA-II | GPS | Rolling Mach. |
|---|---|---|---|
| NSGA-II | – | **0.0057** | **0.0001** |
| GPS | – | – | 0.9205 |

The differences in bold are statistically significant at 0.05 level, even after Bonferroni adjustment (adjusted $\alpha = 0.05/3 = 0.0167$)

## 7 Conclusions

There are legal issues concerning the use of GPS devices for verifying taximeters. However, in our opinion a GPS is the measuring device that best balances cost and accuracy for a VTSS.

To homologate a GPS for this application, we need to guarantee that the tolerance of the measurements is lower than the legal 10% margin. We can not assert that this tolerance holds in absolute terms, but in this paper we have defined how to compute the upper bound of any trajectory length compatible with GPS data, which effectively is a computation of the upper tolerance of the device, for a particular route. These calculations must be repeated each time a taxi is verified, because the obtained margins depend on the GPS signal reception, the satellite configuration and the shape of the path. We have also found that the stronger the turns in the calibration trajectory, the less accurate the measurement is. Therefore, we recommend to avoid convoluted paths in the GPS-based verification of taximeters.

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## References


Abbasbandy S, Asady B (2006) Ranking fuzzy numbers by sign distance. Inform Sci 176:2405–2416

Anile AM, Falcidieno B, Gallo G, Spagnuolo M, Spinello S (2000) Modeling uncertain data with fuzzy B-splines. Fuzzy Sets Syst 113:397–410

Berman P, Schnitger G (1992) On the complexity of approximating the independent set problem. Inform Comput 96:77–94

Buzer L (2009) Optimal simplification of polygonal chains for subpixel-accurate rendering. Comput Geom 42(1):45–59

Casillas J, Cordón O, del Jesus MJ, Herrera F (2001) Genetic tuning of fuzzy rule-based systems integrating linguistic hedges. In: Proceedings of the 9th IFSA world congress and 20th NAFIPS international conference, Vancouver, pp 1570–1574

Chen DZ, Daescu O, Hershberger J, Kogge PM, Mi N, Snoeyink J (2005) Polygonal path simplification with angle constraints. Comput Geom 32(3):173–187

Couso I, Montes S, Gil P (2001) The necessity of the strong alpha-cuts of a fuzzy set. Int J Uncertain Fuzziness Knowl Based Syst 9(2):249–262

Cressie NAC (1991) Statistics for spatial data. Wiley, London. ISBN 0-471-84336-9

de Boor C (1972) On calculating with B-splines. J Approx Theory 6:50–62

Deb K, Goel T (2001) Controlled elitist non-dominated sorting genetic algorithms for better convergence. In: Zitzler E, Deb K, Thiele L, Coello CAC, Corne D (eds) First international conference on evolutionary multi-criterion optimization. Springer, Heidelberg, LNCS No. 1993, pp 67–81

Deb K, Agrawal S, Pratab A, Meyarivan T (2000) A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In: Schoenauer M, Deb K, Rudolph G, Yao X, Lutton E, Merelo JJ, Schwefel HP (eds) Proceedings of the parallel problem solving from nature VI conference. Springer, Heidelberg, pp 849–858

Drysdale RLS, Rote G, Sturm A (2008) Approximation of an open polygonal curve with a minimum number of circular arcs and biarcs. Comput Geom 41(1–2):31–47

Dussault C, Courtois R, Ouellet JP, Huot J (2001) Influence of satellite geometry and differential correction on GPS location accuracy. Wildl Soc Bull 29(1):171–179

Estkowski R, Mitchell JSB (2002) Simplifying a polygonal subdivision while keeping it simple, SCG '01. In: Proceedings of the seventeenth annual symposium on computational geometry, ISBN 1-58113-357-X, 40-49, ACM Press, New York

Ganesan K, Veeramani P (2006) Fuzzy linear programming with trapezoid fuzzy numbers. Ann Oper Res 143:305–315

Goodman IR, Nguyen HT (1985) Uncertainty models for knowledge-based systems. Elsevier, New York

Grewal MS, Weill LR, Andrews AP (2007) Global positioning systems, inertial navigation, and integration, 2nd edn. Wiley, London. ISBN: 978-0-470-04190-1.

Gudmundsson J, Narasimhan G, Smid M (2007) Distance-preserving approximations of polygonal paths. Comput Geom 36(3):183–196

Hershberger J, Snoeyink J (1992) Speeding up the douglas-peucker line simplification algorithm. In: Proceedings of the 5th international symposium on spatial data handling, Charleston, pp 134–143

Hofmann-Wellenhof B, Collins L (2004) GPS theory and practice. Springer, Heidelberg, ISBN 3211835342

Hsu JC (1996) Multiple comparisons: theory and methods. Chapman and Hall, London

Jahanshahloo GR, Soleimani-damaneh M, Nasbaradi E (2004) Measure of efficiency in DEA with fuzzy input-output levels: a methodology for assessing, ranking and imposing of weight restrictions. Appl Math Comput 156:175–187

Kiyota T, Tsuji Y, Kondo E (2000) New multiobjective fuzzy optimization method and its application. In: American control conference ACS'2000 6, Chicago, pp 4224–4228

Lachapelle G, Ryan S (2000) Statistical reliability measures for GPS. In: IMA workshop on mathematical challenges in GPS University of Minnesota, August 16–18

Langley RB (1991) The mathematics of GPS. GPS World 2(7):45–59

Langley RB (1999) Dilution of precision. GPS World 10(5):52–59

Li L, Chen W (1999) Corner detection and interpretation on planar curves using fuzzy reasoning. IEEE Trans Pattern Anal Mach Intell 21(11):1204–1210

Mitchell HB (2006) Ranking type-2 fuzzy numbers. IEEE Trans Fuzzy Syst 14(2):287–294

Mosavi MR, Mohammadi K, Refan MH (2002) Fuzzy processing on GPS data to improve positioning accuracy, before and after S/A Is turned off. In: The Asian GPS conference, India, pp117–120

Naranjo J, González C, García R, de Pedro T (2007) Fuzzy logic for transportation guidance: developing fuzzy controllers for maintaining an inter-vehicle safety headway. In: Bai Y, Zhuang H, Wang D (eds) Advances in industrial control, advanced fuzzy logic technologies in industrial applications. Springer, London, pp 129–143

Otero J, Sanchez L, Alcala-Fdez J (2008) Fuzzy-genetic optimization of the parameters of a low cost system for the optical measurement of several dimensions of vehicles. Soft Comput 12(3):751–764

Quddus MA, Noland RB, Ochieng WY (2006) A high accuracy fuzzy logic based map matching algorithm for road transport. J Intell Transport Syst 10(3):103–115

Sánchez L, Couso I (2007) Advocating the use of imprecisely observed data in genetic fuzzy systems. IEEE Trans Fuzzy Syst 15(4):551–562

Sánchez L, Couso I, Casillas J (2006) A multiobjective genetic fuzzy system with imprecise probability fitness for vague data. In: Second international symposium on evolving fuzzy systems 2006 (EFS06), Ambleside Lake District, pp 131–137

Sánchez L, Couso I, Casillas J (2007) Modeling vague data with genetic fuzzy systems under a combination of crisp and imprecise criteria. In: First IEEE symposium on computational intelligence in multi-criteria decision-making (MCDM 2007), Honolulu

Sánchez L, Couso I, Casillas J (2009) Genetic learning of fuzzy rules based on low quality data. Fuzzy Sets Syst (submitted)

Sheen JN (2006) Fuzzy evaluation of cogeneration alternatives on a petrochemical industry. Comput Math Appl 49:741–755

Strang G, Borre K (1997) Random variables and covariance matrices. In: Linear algebra, geodesy, and GPS. Wellesley-Cambridge Press, Massachusetts

Snyder JP (1982) Map projections used by the U.S. geological survey. Geol. Survey Bulltein 1532. U.S. Government Printing Office, Washington

Sun H, Wu J (2006) A new approach for ranking fuzzy numbers based on fuzzy simulation analysis method. Appl Math Comput 174:755–767

Tran L, Duckstein L (2002) Comparisions of fuzzy numbers using a fuzzy distance measure. Fuzzy Sets Syst 130:321–341

Trebi-Ollennu A, White BA (1997) Multiobjective fuzzy genetic algorithm optimization approach to nonlinear control system design. IEEE Proc Control Theory Appl 2(144):137–142

van Diggelen F (2007) GNSS accuracy: Lies, Damn Lies, and statistics. GPS World 9(1):41–45

Wang Y-M, Greatbanks R, Yang J-B (2005) Interval efficiency assessment using data envelopment analysis. Fuzzy Sets and Syst 153:347–370

Yeh C-H, Deng H (2004) A practical approach to fuzzy utilities comparision in fuzzy multicriteria analysis. Int J Approx Reason 35:179–194

Youssef H, Sait SM, Khan A (2000) Fuzzy simulated evolution algorithm for topology design on campus networks. In: Congress on evolutionary computation 1, Piscataway, pp 180–187

Sun Z, Dong T, Liao X, Zhang R, Song D (2007) Fuzzy logic for flight control II: fuzzy logic approach to path tracking and obstacles avoidance of UAVs. In: Bai Y, Zhuang H, Wang D (eds) Advances in industrial control, advanced fuzzy logic technologies in industrial applications. Springer, London, pp 223–235

Zheng G, Zhao R, Guo Q, Liu L, Bai L, Du L (2006) Adaptive avoiding divergence algorithm for GPS/DR integration based on fuzzy logic. Intelligent control and automation, 2006. WCICA 2006. In: The sixth world congress, vol 2, 21–23 June 2006, pp 8679–8682